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Gait & Posture 22 (2005) 182–188

GAIT & POSTURE

www.elsevier.com/locate/gaitpost

Assessment of two-dimensional induced accelerations from measured kinematic and kinetic data

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Received 29 January 2004; accepted 8 August 2004

Abstract

A simple algorithm is presented to calculate the induced accelerations of body segments in human walking for the sagittal plane. The method essentially consists of setting up 2×4 force equations, 4 moment equations, 2×3 joint constraint equations and two constraints related to the foot-ground interaction. Data needed for the equations are, next to masses and moments of inertia, the positions of ankle, knee and hip. This set of equations is put in the form of an 18×18 matrix or 20×20 matrix, the solution of which can be found by inversion. By applying input vectors related to gravity, to centripetal accelerations or to muscle moments, the 'induced' accelerations and reaction forces related to these inputs can be found separately. The method was tested for walking in one subject. Good agreement was found with published results obtained by much more complicated three-dimensional forward dynamic models.

Keywords: Forward dynamics; Induced accelerations; Induced power; Sagittal plane

1. Introduction

In a number of recent papers the knowledge of the mechanics of human walking has considerably been increased [1–5]. These papers are based on complicated three-dimensional forward dynamics models including many muscles. Some of the methodological innovations are the concept of induced accelerations [6], the related power flows [3], and the subdivision of the ground reaction force into contributions of the separate muscles [5]. Induced accelerations are the accelerations caused by a single source, e.g. the force of one muscle, the weight of one or more segments, etc.

A problem is that these forward dynamics models are very complicated to set up, to run, and most of all, to have them find an optimal solution. Because of this, the implementation and maintenance of such a model is at present reserved to research groups which are able to invest many years of effort in the purpose, and will be out of reach to most clinical gait laboratories. This is unfortunate, because in clinical groups there is a daily interest in the question of the action of muscles: what is the consequence of some (unusual) muscle activity in a certain phase of (pathological) gait. Induced accelerations seem very promising for this aim as they answer the question which is the part of the acceleration of a body segment and is caused by the moments due to a single muscle.

The aim of this short paper is to show that assessment of induced accelerations in a sagittal plane is possible with only basic kinematic data: the *x*- and *y*-coordinates of the ankle, knee, hip and shoulder, data which are readily available from standard optoelectronic techniques or video registrations. Force plate data are helpful, but not strictly necessary. One of the reasons that this analysis is relatively simple, lies in the fact that calculation of the induced accelerations is only the first step in a forward dynamics model. The much more delicate steps of the integration of accelerations to positions and the need to arrive at stable movements, which requires that all relevant passive and muscular forces are included in the model, and which leads to very extensive optimization procedures, are not needed.

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doi:10.1016/j.gaitpost.2004.08.007

Nomenclature

$\boldsymbol{a} = (a_x, a_y)$	acceleration vector (2D)
$\mathbf{A}$	matrix of equations of motion, see (1)
$\mathbf{c}$	vector of inputs, see Table 2
$\boldsymbol{F} = (F_x, F_y)$	force at joint, from distal on proximal segment
I	moment of inertia
m	mass of a segment
M	joint moment
$\boldsymbol{r} = (x, y)$	position vector (2D)
$\mathbf{v}$	velocity of the CoM of a segment
$\mathbf{x}$	vector of outputs, see columns of **A** in Table 1
Greek letters	
α	angular acceleration of segment
ω	angular velocity of segment
Subscripts	
ankle, knee, hip	relating to joint or joint centre
foot, shank, thigh, HAT	relating to (centre of mass of) segment HAT = head-arms-trunk, considered as one segment

In most papers the forward dynamics models are formulated in terms of generalised coordinates [1]. For three-dimensional models this is sensible, as it greatly reduces the number of equations of movement. A disadvantage is that the relation between the generalised coordinates of the dynamics and the rectangular coordinates, in which kinematic data are usually represented, can be rather complicated. In a previous paper of one of us [7] it was shown that, for a simple two-dimensional model, the equations of motion can be written and solved in the usual rectangular coordinates. The present paper is an elaboration of this model suitable for human walking.

2. Methods

2.1. The matrix A

The two-dimensional one-leg sagittal plane model of human walking, to be presented consists of four segments: foot, shank, thigh and head-arms-trunk (HAT), linked at three joints: ankle, knee and hip. Moments M due to muscle forces act at the three joints; a moment $-M$ acts on the proximal segment, and $+M$ on the distal segment. Also at each joint acts an intersegmental force. By definition this force acts from the distal on the more proximal segment, e.g. the force at the ankle acts from the foot on the shank.

The essential point of the proposed method is that the equations of motion are written in matrix form:

$$\mathbf{A}\mathbf{x} = \mathbf{c} \tag{1}$$

in which $\mathbf{x}$ is a vector of unknowns and $\mathbf{c}$ a vector of known quantities. This set of equations is solved by inverting matrix $\mathbf{A}$:

$$\mathbf{x} = \mathbf{A}^{-1}\mathbf{c} \tag{2}$$

yielding the vector of unknowns $\mathbf{x}$. This vector consists of (see columns of Table 1), respectively, the intersegmental forces at ankle, knee and hip, x- and y-components (1–6), the segment accelerations of foot, shank, thigh and HAT segment (7–14), the angular accelerations of the four segments (15–18) and, finally the components of the ground reaction forces (19–20). The input vector $\mathbf{c}$ contains, among other things, segment weights and joint moments (Table 2).

The matrix **A** is given in Table 1a, with some submatrices in Table 1b–d. The first eight lines represent the force (Newton's) equations for the four segments. For e.g. the shank, lines three and four, this equation holds:

$$\boldsymbol{F}_{\text{ankle}} - \boldsymbol{F}_{\text{knee}} + m_{\text{shank}}\boldsymbol{g} = m_{\text{shank}}\boldsymbol{a} \tag{3}$$

Lines 15–18 contain the moment (Euler's) equations for the four segments. For the shank again:

$$(\boldsymbol{r}_{\text{ankle}} - \boldsymbol{r}_{\text{shank}}) \times \boldsymbol{F}_{\text{ankle}} - (\boldsymbol{r}_{\text{knee}} - \boldsymbol{r}_{\text{shank}}) \times \boldsymbol{F}_{\text{knee}} - M_{\text{ankle}} + M_{\text{knee}} = I_{\text{shank}}\alpha_{\text{shank}} \tag{4}$$

The coupling of the segments in the three joints is represented in 3×2 constraint equations, lines 9–14. They express that the joints are points common to two solid segments, that therefore the acceleration of the joint should be identical when expressed in the accelerations of both adjoining segments. For e.g. the ankle, lines 9 and 10, it should hold that

$$\boldsymbol{a}_{\text{foot}} + \alpha_{\text{foot}} \times (\boldsymbol{r}_{\text{ankle}} - \boldsymbol{r}_{\text{foot}}) - \omega^2_{\text{foot}}(\boldsymbol{r}_{\text{ankle}} - \boldsymbol{r}_{\text{foot}}) = \boldsymbol{a}_{\text{shank}} + \alpha_{\text{shank}} \times (\boldsymbol{r}_{\text{ankle}} - \boldsymbol{r}_{\text{shank}}) - \omega^2_{\text{shank}}(\boldsymbol{r}_{\text{ankle}} - \boldsymbol{r}_{\text{shank}}) \tag{5}$$

Running over the complete matrix **A** thus derived, we see that the part depicted in Table 1a, up to line 18, is constant. The two submatrices **A** (15:18,1:6) and **A** (9:14,15:18), given in Table 1b and c, on the other hand, depend on the coordinates of the joints and the centres of mass of the segments, and thus change with the movement.

2.2. The foot

In the swing phase the foot is free from external forces, and the above equations **A** (1:18,1:18) completely describe the dynamics in that case. In stance there is a ground reaction force acting on the foot at a variable point on the ground, the centre of pressure (CoP). For stance, therefore, the foot segment was modelled as being jointed in the CoP with the ground. The required equations were put in lines and columns 19 and 20 of matrix **A**.

Table 1
System matrix **A**: (a) overview with all constant factors; (b) submatrix **A** (15:18,1:6); (c) submatrix **A** (9:14,15:18); (d) nonzero part of columns 19 and 20

(a)	Ankle		Knee		Hip		Foot		Shank		Thigh		HAT		Foot	Shank	Thigh	HAT
	F_x 1	F_y 2	F_x 3	F_y 4	F_x 5	F_y 6	a_x 7	a_y 8	a_x 9	a_y 10	a_x 11	a_y 12	a_x 13	a_y 14	α 15	α 16	α 17	α 18
Foot x 1	−1						$-m_{\text{foot}}$											
y 2		−1						$-m_{\text{foot}}$										
Shank x 3	1		−1						$-m_{\text{shank}}$									
y 4		1		−1						$-m_{\text{shank}}$								
Thigh x 5			1		−1						$-m_{\text{thigh}}$							
y 6				1		−1						$-m_{\text{thigh}}$						
HAT x 7					1								$-m_{\text{HAT}}$					
y 8						1								$-m_{\text{HAT}}$				
Ankle x 9							1		−1						See Table 1c			
y 10								1		−1								
Knee x 11									1		−1							
y 12										1		−1						
Hip x 13											1		−1					
y 14												1		−1				
Foot 15	See Table 1b														$-I_{\text{foot}}$			
Shank 16																$-I_{\text{shank}}$		
Thigh 17																	$-I_{\text{thigh}}$	
HAT 18																		$-I_{\text{HAT}}$
CoP x 19							1								$-(y_{\text{CoP}} - y_{\text{foot}})$			
CoP y 20								1							$(x_{\text{CoP}} - x_{\text{foot}})$			

(b)	1	2	3	4	5	6
Foot 15	$(y_{\text{ankle}} - y_{\text{foot}})$	$-(x_{\text{ankle}} - x_{\text{foot}})$				
Shank 16	$-(y_{\text{ankle}} - y_{\text{shank}})$	$(x_{\text{ankle}} - x_{\text{shank}})$	$(y_{\text{knee}} - y_{\text{shank}})$	$-(x_{\text{knee}} - x_{\text{shank}})$		
Thigh 17			$-(y_{\text{knee}} - y_{\text{thigh}})$	$(x_{\text{knee}} - x_{\text{thigh}})$	$(y_{\text{hip}} - y_{\text{thigh}})$	$-(x_{\text{hip}} - x_{\text{thigh}})$
HAT 18					$-(y_{\text{hip}} - y_{\text{HAT}})$	$(x_{\text{hip}} - x_{\text{HAT}})$

(c)	15	16	17	18
Ankle x 9	$-(y_{\text{ankle}} - y_{\text{foot}})$			
y 10	$(x_{\text{ankle}} - x_{\text{foot}})$	$-(x_{\text{ankle}} - x_{\text{shank}})$		
Knee x 11		$-(y_{\text{knee}} - y_{\text{shank}})$	$(y_{\text{knee}} - y_{\text{thigh}})$	
y 12		$(x_{\text{knee}} - x_{\text{shank}})$	$-(x_{\text{knee}} - x_{\text{thigh}})$	
Hip x 13			$-(y_{\text{hip}} - y_{\text{thigh}})$	$(y_{\text{hip}} - y_{\text{HAT}})$
y 14			$(x_{\text{hip}} - x_{\text{thigh}})$	$-(x_{\text{hip}} - x_{\text{HAT}})$

(d)	F_e x 19	F_e y 20
Foot x 1	1	
y 2		1
15	$-(y_{\text{CoP}} - y_{\text{foot}})$	$(x_{\text{CoP}} - x_{\text{foot}})$

All zero entries are left blank.

When inverting **A**, in stance the full 20 × 20 matrix is inverted. For the swing period only the submatrix **A** (1:18,1:18) is inverted, and the remaining lines and columns 19 and 20 of $\mathbf{A}^{-1}$ are made zero.

2.3. *The input vector* c

The composition of **c**, the vector of known variables, is given in Table 2. The first six components are (minus the) forces acting on the centres of mass of the various segments. In general this will be only gravity. To the HAT segment the force from the contralateral leg can be added, which can be obtained from inverse dynamics, e.g. by shifting data from the ipsilateral leg over half a stride. (For inverse dynamics force plate data should be available.) The components 9–14 comprise the centripetal accelerations relevant for the three joints, which involve the angular velocities of the segments. Components 19–20 are similar terms related to foot movement.

Table 2
The vector of inputs c

1	0	gravity
2	$9.81\ m_{\text{foot}}$	
3	0	
4	$9.81\ m_{\text{shank}}$	
5	0	
6	$9.81\ m_{\text{thigh}}$	
7	$0 + F_{\text{HATx}}$	+ force contralateral leg
8	$9.81\ m_{\text{HAT}} + F_{\text{HATy}}$	+ force contralateral leg
9	$-\omega^2_{\text{shank}}(x_{\text{ankle}} - x_{\text{shank}}) + \omega^2_{\text{foot}}(x_{\text{ankle}} - x_{\text{foot}})$	centripetal accelerations
10	$-\omega^2_{\text{shank}}(y_{\text{ankle}} - y_{\text{shank}}) + \omega^2_{\text{foot}}(y_{\text{ankle}} - y_{\text{foot}})$	
11	$-\omega^2_{\text{thigh}}(x_{\text{knee}} - x_{\text{thigh}}) + \omega^2_{\text{shank}}(x_{\text{knee}} - x_{\text{shank}})$	
12	$-\omega^2_{\text{thigh}}(y_{\text{knee}} - y_{\text{thigh}}) + \omega^2_{\text{shank}}(y_{\text{knee}} - y_{\text{shank}})$	
13	$-\omega^2_{\text{HAT}}(x_{\text{hip}} - x_{\text{HAT}}) + \omega^2_{\text{thigh}}(x_{\text{hip}} - x_{\text{thigh}})$	
14	$-\omega^2_{\text{HAT}}(y_{\text{hip}} - y_{\text{HAT}}) + \omega^2_{\text{thigh}}(y_{\text{hip}} - y_{\text{thigh}})$	
15	$-M_{\text{ankle}}$	muscle moments
16	$M_{\text{ankle}} - M_{\text{knee}}$	
17	$M_{\text{knee}} - M_{\text{hip}}$	
18	$+M_{\text{hip}}$	
19	$\omega^2_{\text{foot}}(x_{\text{foot}} - x_{\text{CoP}})$	foot centripetal acc.
20	$\omega^2_{\text{foot}}(y_{\text{foot}} - y_{\text{CoP}})$	

Lines 1–8: gravity. If wanted, any force acting at the centre of mass of a segment can be added, e.g. the force from the contralateral leg to the HAT segment in lines 7 and 8. Lines 9–14: centripetal accelerations around ankle, knee and hip. Lines15–18: muscle moments. Lines 19 and 20: centripetal acceleration of foot around the CoP (only used in stance).

Components 15–18 are combinations of the three joint moments and thus represent the muscular input to the movement. The moments can be expressed as the product of the muscle force **f** times a matrix of moment arms **R**, dependent on the joint angles [1,8]:

$$\mathbf{m} = \mathbf{R}(\theta)\mathbf{f} \tag{6}$$

in which $\mathbf{m} = (M_{\text{ankle}}, M_{\text{knee}}, M_{\text{hip}})^{\text{T}}$.

It is important to notice that the matrix multiplication (2) is a linear operation. This means that the solution to an input $(\mathbf{c}_1 + \mathbf{c}_2)$ is equal to the sum of the separate solutions:

$$\mathbf{A}^{-1}(\mathbf{c}_1 + \mathbf{c}_2) = \mathbf{A}^{-1}\mathbf{c}_1 + \mathbf{A}^{-1}\mathbf{c}_2 \tag{7}$$

A consequence is that induced forces and accelerations due to gravity, to centripetal accelerations or to the force of a single muscle can be separately obtained, simply by inserting a single set of quantities in the input vector, leaving all other entries zero.

2.4. Power and work

It can be seen from (3) that mass × the induced acceleration equals the sum of forces acting on a segment. As a consequence the power from all forces done on the segments is equal to

$$P_1 = m_{\text{segment}}(\boldsymbol{a}_{\text{induced}} \cdot \boldsymbol{v}_{\text{CoM}}) \tag{8}$$

Similarly the moment equation (4) gives for the power related to rotation

$$P_2 = I_{\text{segment}}(\alpha_{\text{segment}} \cdot \omega_{\text{segment}}) \tag{9}$$

The total power done on the segment is the sum of P_1 and P_2. The velocities $\boldsymbol{v}_{\text{segment}} = (v_x, v_y)$ and ω_{segment} can be calculated from the kinematic data.

2.5. Experimental data

The described method will be illustrated by a registration of walking. The subject was a 22 year male, stature 1.82 m, mass 73 kg, walking at 1.25 m s^{-1}. Kinematic data were recorded by an Optotrak optoelectronic system from infrared emitting active markers attached at MTP5, ankle, knee, hip, and shoulder at points specified by Winter [9]. Ground reaction force was recorded from a Bertec 4060 force plate. Centre of mass positions and joint moments according to inverse dynamics [10] were calculated on the basis of anthropometric data from Winter [9]. Sample frequency was 100 Hz. Although three-dimensional data were recorded, only data from the sagittal (XY) plane were used in the analysis. It was ensured that the progression was indeed in this plane. The position of the CoP was calculated from the force plate data.

3. Results

As an example of what can be calculated with this method, some data on soleus and gastrocnemius in walking will be shown. It was assumed that both parts of triceps surae shared equal (50%) parts of the total ankle moment, as calculated by inverse dynamics. For gastrocnemius, a ratio

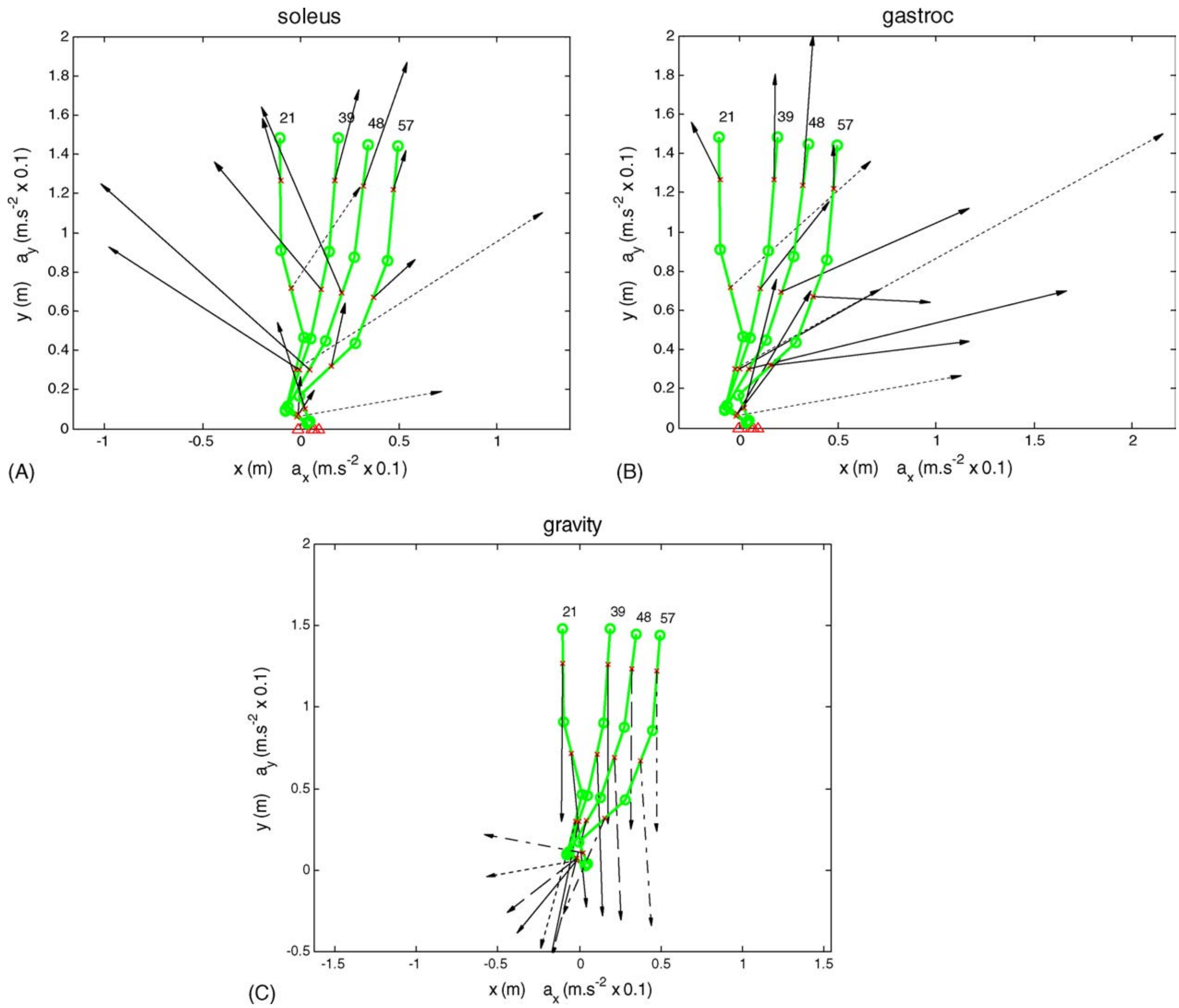

Fig. 1. Stick diagram with induced accelerations (arrows) for a subject walking 1.25 m s^{-1} at four instants during stance, at 21% (- - -) 39% (–), 48% (– – –) and 57% (-·-·) of the gait cycle. Induced accelerations originate in the centre of mass (×) of each segment. Joint markers are, from top to bottom: shoulder, hip, knee, ankle, and MTP5. (A) soleus, (B) gastrocnemius, (C) gravity. In (A) and (B) also the positions of the CoP (triangles) have been depicted.

of 1:2 was assumed for the moment arms with respect to knee and ankle.

Fig. 1a shows the vectors of the accelerations induced by soleus, Fig. 1b those of gastrocnemius, in four stick diagrams at 21%, 39%, 48% and 57% of the gait cycle. Stance was from 0% to 61% of the cycle. The ankle moments were −40, −88, −137 (maximum) and −45 Nm, respectively, at these points in the cycle. In addition, the accelerations induced by gravity are shown in Fig. 1c. The power curves for soleus and gastrocnemius are given in Fig. 2a and b. In both curves power has been divided into the power to the HAT segment, both horizontal, vertical and rotational, and the power to the leg, the sum of foot, shank, and thigh. As a comparison muscle power, $P_m = \sum M_{joint}\omega_{joint}$ has been shown as well. It is seen that P_m is closely equal to the sum of P_{HAT} and P_{leg} (maximum difference 20 W).

4. Discussion

4.1. Actions of soleus and gastrocnemius

The present results may serve as an example of the kind of results that can be obtained. Remarkable is the difference in the actions of mono-articular soleus and bi-articular gastrocnemius, an effect we had not taken into account in earlier work [11]. Soleus adds to the propulsion of the trunk, as the induced acceleration of the HAT has a forward component during the greater part of stance which gastrocnemius has not. In contrast, gastrocnemius induces marked forward acceleration to the leg, which soleus does only in part of stance. Comparison with the induced accelerations due to gravity shows that both muscles have an important support (anti-gravity) function. The results on induced accelerations are in agreement with those of Zajac

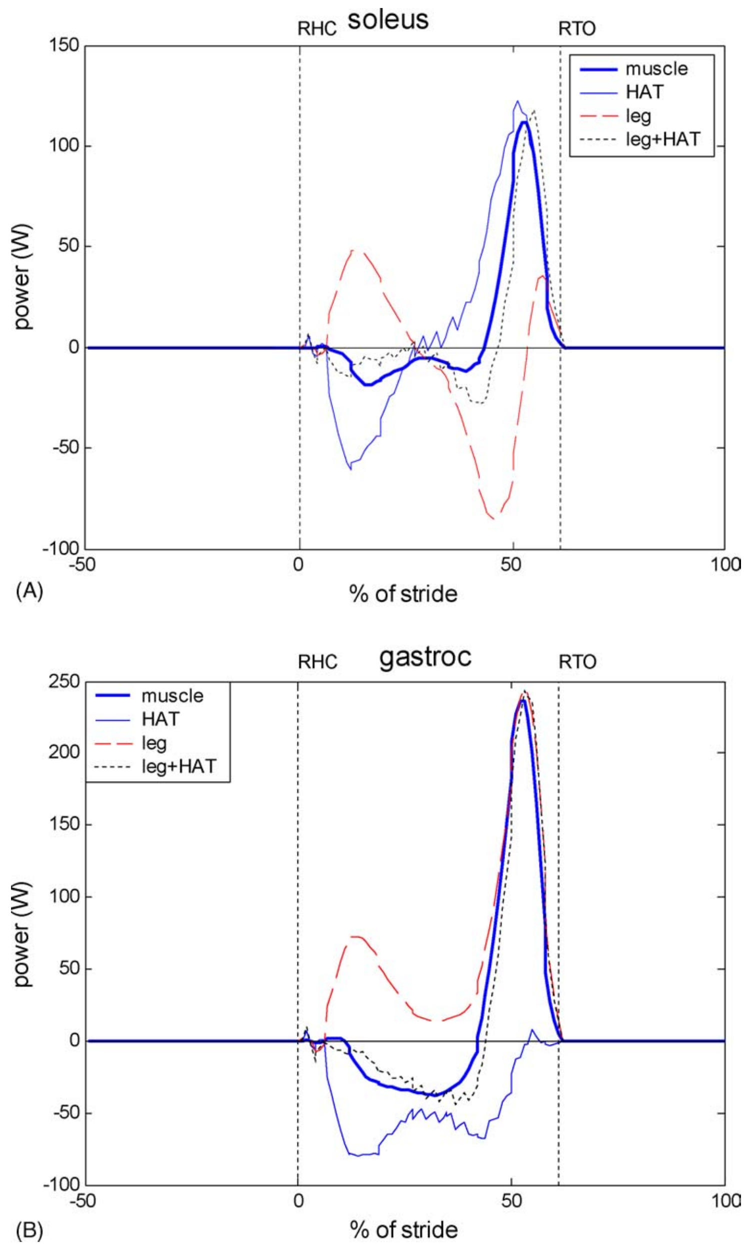

Fig. 2. Power delivered to leg and trunk (HAT), calculated from induced accelerations for soleus (A) and gastrocnemius (B). Same recording as Fig. 1.

[4], compare the present Fig. 1a and b with their Fig. 4. The power curves Fig. 2, can be compared to Neptune's results [3] in their Fig. 6. The differences in function between soleus and gastrocnemius are extensively discussed in the latter paper.

A comparison of Fig. 1a–c shows that induced accelerations should be interpreted with care. At 39% and 48% of stance, e.g., soleus induces marked backward accelerations in shank and thigh, Fig. 1a. These do not result in a backward movement however, they counteract forward accelerations due to gastrocnemius, Fig. 1b, and together these two muscles yield an upward acceleration of trunk and leg segments that counteracts the mainly downward accelerations due to gravity, Fig. 1c. The gravity-induced accelerations depict what would happen if only gravity acted with no moments acting around the joints: the leg would collapse under the weight of the trunk. The ankle moment from soleus and gastrocnemius (in combination with the knee extension moment from quadriceps) prevents this collapse.

In the interpretation, an analysis of the power as in Fig. 2 may be helpful. In the power, the velocity as derived from the kinematic data is taken into account, as well as, the mass of the segment. In this way the division of muscle power over the segments can be analysed quantitatively.

It is seen that there is a good match between muscle power and the total power calculated from the induced accelerations. This appeared to be related to the modelling of the foot ground interaction, a well-known problem in forward dynamics. In the present model the foot is modelled as having a 'joint' at the CoP with the ground, thus at a position that changes during the roll-off of the foot. This forces the model ground reaction force to be at its measured position, the CoP. In an earlier version of the model, we modelled the foot for the first part of stance as being rigidly connected to the floor, and after heel rise as jointed at the MTP5 joint. This model yielded much less accurate results, with errors up to 120 W (50%). Another check of the model is to use total ankle, knee and hip moments obtained from inverse dynamics as input and to compare the calculated ground reaction force with the measured data. Here again, the agreement was very good for the CoP-model: the error was less than 10 N (1%) except during the initial 100 ms after heel contact. With the earlier model this was much worse. Only in the recent paper of Schwartz and Lakin [12] we found the same approach.

4.2. Properties of induced accelerations

An important property of dynamics was already mentioned in (7), the linearity of the relation between muscle force and induced acceleration [1,13]. One effect was that the accelerations induced by different muscles or other factors can be separated. A second effect is that the *direction* of the induced accelerations and reaction forces is *independent of the magnitude* of the input moment or force [8]. The only relevant factors are the relative positions of the limbs, the way the foot interacts with the ground and, in bi-articular muscles, the ratio between the moment arms. Accelerations from a set of muscles can just be obtained by vector addition of the respective induced accelerations. To a certain extent this may even hold for the displacement: the direction of the induced acceleration is only dependent on the relative position, except for the effect of the centripetal accelerations, which are relatively small as long as the rotations are not too fast. In our opinion these effects are of major interest for motor control.

4.3. Practical aspects

The proposed algorithm [7] is rather brute force; all relevant equations are just put below each other and solved as a whole. In the present context this has the advantage that setting up the equations is straightforward and transparent. A possible objection might have been practical problems in inverting large matrices. With present-day PC's this is no problem, however. On a standard Pentium V PC, with Windows and MatLab, 100 matrix inversions took only 90 ms. Even implementation as an Excel worksheet has proven possible.

The main difference between the present simple model and the models of the Zajac group [1–4], is, that it does not attempt to predict the movement from the input muscle forces, but uses measured kinematic data instead. This obviates the extensive optimization process needed to arrive at muscle forces or activations that result in sensible movements. This holds similarly for the use of the measured CoP as the point of application of the ground reaction force. Forward dynamics models of locomotion require quite complicated foot models to obtain e.g. reasonable ground reaction forces [5]. Although the present two-dimensional model is definitively simpler than existing three-dimensional models [1–5,12], comparison with published results of these models has shown essentially the same outcome.

As in model inputs mainly joint positions are needed and the position of the CoP. The magnitude of the muscle force or moment is not needed if only the directions of the accelerations are of interest. Angular velocities are only needed when the effects of centripetal accelerations are to be studied. A possible application of the described simple method may be in the context of interpretation of EMG patterns; if a muscle shows activity in some stage of a movement, what is the effect of it?

Acknowledgements

A set of MatLab® programs and an EXCEL®-worksheet related to this paper can be obtained from the ISB website: http://www.isbweb.org/software/index.html.

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